INTERVAL NOTATION

❑ *INTRODUCTION*

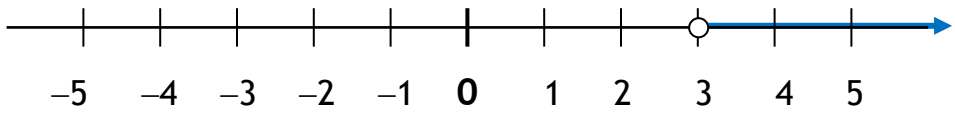
❑ *EXAMPLES*

First Example: Consider all the real numbers greater than 3. One simple way to express this set of numbers is the following ***inequality***:

$$x > 3$$

Do you see that this inequality can also be written "3 < x"?

We can also ***graph*** this set of numbers on a number line:



Notice that we put an "open dot" at $x = 3$ to show that the 3 is not part of the set of numbers. But the arrow goes infinitely to the right because $x > 3$ is the set of numbers greater than 3.

And a third way to denote this interval is called ***interval notation***, and for this inequality, we write:

$$(3, \infty)$$

Note that the numbers 3.01, π, 17, and 200 are part of the set; but the numbers –5, 0, 2.5, and 3 are not part of the set.

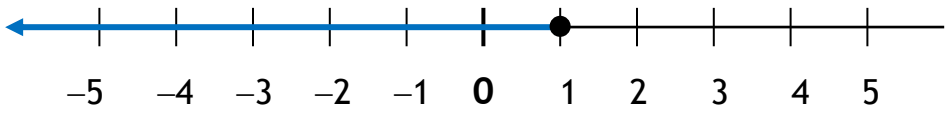
The parenthesis next to the 3 is analogous to the "open dot" on the graph — it means exclude the endpoint. And you always use a parenthesis at the ∞ or –∞ end of an interval because ∞ is not really a number, so you can't possibly include it.

Second Example: Now we consider all the real numbers less than or equal to 1. This set of numbers can be written as an inequality like this:

$$x \leq 1$$

Note that this inequality can also be expressed as: $1 \geq x$

As a graph on a number line, we write:



The "solid dot" is used to show that $x = 1$ is part of the set of the numbers. And since x must be less than or equal to 1, the arrow goes infinitely to the left. Either as an inequality or a graph, you should see that the numbers –3, –1.1, $\frac{7}{8}$, and 1 are part of the set, while the numbers 1.001 and $\sqrt{2}$ are not part of the set.

As for interval notation, we write

$$\mathbf{(-\infty, 1]}$$

The (square) bracket next to the 1 is analogous to the "solid dot" on the graph — it means include the endpoint. And, as mentioned before, always use a parenthesis with $\pm\infty$, since you can't ever "get" to ∞.

Third Example: Now it's time for an interval that represents all the numbers between two numbers. Consider the double inequality

$$-2 \leq x < 5$$

This can be read as "all real numbers between –2 and 5, including the –2, but excluding the 5."

As for interval notation and a graph, here they are:

$$[-2, 5)$$

Note that some numbers in the interval are –2, 0, $\sqrt{24}$, and 4.9999. But the numbers –2.1, 5, and 2π are not in the interval.

Fourth Example: Sometimes answers to an inequality problem end up looking something like this:

$$x < 3 \text{ OR } x \geq 5$$

This means pretty much what it says: x can be less than 3, or it can be greater than or equal to 5. As long as x satisfies (at least) one of the two conditions, it's part of the answer. So some x's that satisfy the inequality are 0, 2.9, 5, and 3π. On the other hand, some numbers that do NOT satisfy it are 3, π, and 4.99.

How do we express this in interval notation? Like this, using the **union** symbol:

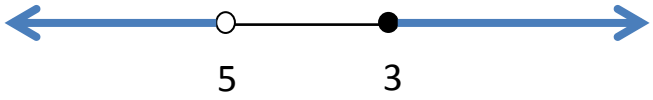
$$(-\infty, 3) \cup [5, \infty)$$

The *union* symbol, $\cup$, just means putting all the elements of the two sets into one big set; for example,

{a, b} $\cup$ {a, c, d} = {a, b, c, d}

Note that the "overlap" of 'a' is listed only once in the final set.

And as a graph, we write:



Can you see that the interval $(-\infty, \infty)$ is the same as the entire set of real numbers?

$$(-\infty, \infty) = \mathbb{R}$$

In fact, using the *union* notation, we can also write

$$\mathbb{R} = \text{Rationals} \cup \text{Irrationals}$$

where the Rational numbers are all decimals that repeat, while the Irrational numbers are the decimals that don't repeat. So $\mathbb{R}$ is simply the set of ALL decimals.

❑ SUMMARY OF INEQUALITIES AND INTERVALS

Inequality	Interval
$x > b$	(b, ∞)
$x \geq b$	$[b, \infty)$
$x < a$	$(-\infty, a)$
$x \leq a$	$(-\infty, a]$
$a \leq x \leq b$ which means $x \geq a$ AND $x \leq b$, which means that x is *between* a and b, including both the a and the b.	$[a, b]$
$x < a$ OR $x > b$	$(-\infty, a) \cup (b, \infty)$

Homework

1. Convert each inequality to *interval notation*:

   a. $x > 2$ b. $x \leq 5$ c. $-1 \leq x < 6$ d. $x < -3$ OR $x > 0$

2. Convert each interval to an *inequality*:

   a. $[3, \infty)$ b. $(-\infty, -5)$ c. $(-1, 8]$ d. $(-\infty, -2] \cup (7, \infty)$

3. What is the relationship between the notation $\mathbb{R}$ and the notation $\mathbf{(-\infty, \infty)}$?

Solutions

1. a. $(2, \infty)$ b. $(-\infty, 5]$ c. $[-1, 6)$ d. $(-\infty, -3) \cup (0, \infty)$

2. a. $x \geq 3$ b. $x < -5$ c. $-1 < x \leq 8$ d. $x \leq -2$ OR $x > 7$

3. They both mean exactly the same thing: the set of all **real numbers**.

"Develop a passion for learning. If you do, you will never cease to grow."

Anthony J. D'Angelo